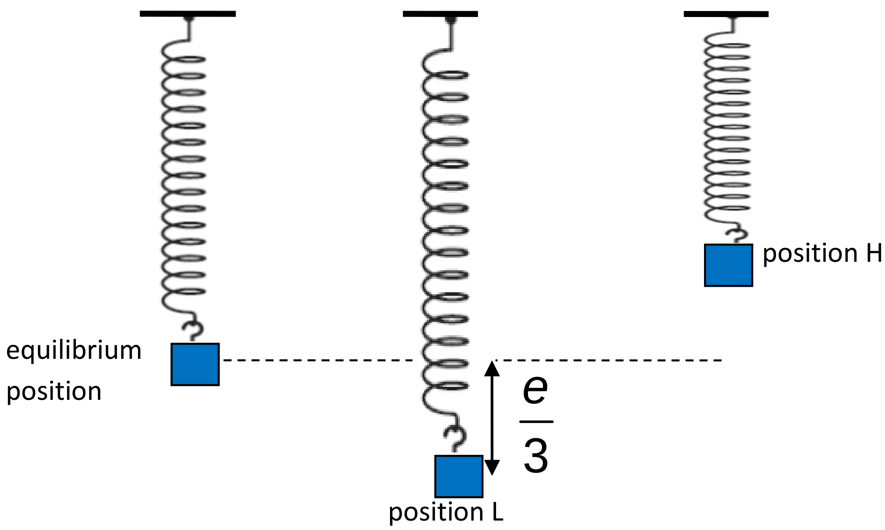
Teacher notes
Topic C

A problem on elastic energy in SHM

A mass hangs in equilibrium at the end of spring. At equilibrium the spring is extended by a distance e. The mass is pulled to position L, a distance $\frac{e}{3}$ below the equilibrium position.



When the mass is released, it performs simple harmonic oscillations between positions L and H with angular frequency ω. The displacement from equilibrium is given by the equation $x = \frac{e}{3}\sin(\omega t + \phi)$. Displacements below the equilibrium position are taken as positive.

(a) Show that $\phi = \frac{\pi}{2}$.

(b) Show explicitly, using the equation for displacement, that position H is a distance $\frac{e}{3}$ above the equilibrium position.

(c) Determine the ratio $\dfrac{\text{elastic potential energy at L}}{\text{elastic potential energy at H}}$.

Answers

(a) At $t = 0$, $\frac{e}{3} = \frac{e}{3}\sin(0+\phi)$ so $\sin\phi = 1$. Hence $\phi = \frac{\pi}{2}$.

(b) H is attained after half a period. $\omega = \frac{2\pi}{T}$ so $x = \frac{e}{3}\sin(\frac{2\pi}{T}\times\frac{T}{2}+\frac{\pi}{2})$ i.e. $x = \frac{e}{3}\sin(\frac{3\pi}{2}) = -\frac{e}{3}$.

The distance is thus $\frac{e}{3}$ above the equilibrium position.

(c) Ratio is $\dfrac{\frac{1}{2}k(e+\frac{e}{3})^2}{\frac{1}{2}k(e-\frac{e}{3})^2} = \left(\dfrac{\frac{4e}{3}}{\frac{2e}{3}}\right)^2 = 4$.